CONJUGATION OF BICOMPLEX MATRIX

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ABSTRACT

In this paper, we have studied Bicomplex matrix with the help of two different idempotent techniques. We have defined three types of conjugation of Bicomplex Matrix also three types of Hermitian and Skew-Hermitian Matrix and obtained some results.

Keywords: Bicomplex numbers, Conjugation of Bicomplex Numbers, Bicomplex Matrix, Hermitian and Skew-Hermitian Matrix

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INTRODUCTION

Throughout this paper, the set of Bicomplex numbers is denoted by C_2 and the sets of complex and real numbers are denoted by C_1 and C_0, respectively. For details of the theory of bicomplex numbers, we refer to [L1], [P1] and [S1].

The set of Bicomplex Numbers defined as:

$$C_2 = \{x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4 : x_1, x_2, x_3, x_4 \in C_0, i_1 \neq i_2 \text{ and } i_1^2 = i_2^2 = -1,\ i_1 i_2 = i_2 i_1\}$$

We shall use the notations $C(i_1)$ and $C(i_2)$ for the following sets:

$$C(i_1) = \{u + i_1 v : u, v \in C_0\} \quad C(i_2) = \{\alpha + i_2\beta : \alpha, \beta \in C_0\}$$

The set of Hyperbolic Numbers H, defined as

$$H = \{x + y i_1 i_2 : x, y \in C_0\}$$

1.1 Idempotent Elements:

Besides 0 and 1, there are exactly two non – trivial idempotent elements in C_2, denoted as e_1 and e_2 and defined as $e_1 = \frac{1 + i_1i_2}{2}$ and $e_2 = \frac{1 - i_1i_2}{2}$

Note that $e_1 + e_2 = 1$ and $e_1e_2 = e_2e_1 = 0.$

1.2 Cartesian idempotent set:

Cartesian idempotent set X determined by X_1 and X_2 is denoted as $X_1 \times_e X_2$ and is defined as

$$X = X_1 \times_e X_2 = \left\{ \xi \in C_2 : \xi = {}^1\xi\, e_1 + {}^2\xi\, e_2, \left({}^1\xi, {}^2\xi\right) \in X_1 \times X_2 \right\}$$

$$C_2 = C(i_1) \times_e C(i_1) = C(i_1)e_1 + C(i_1)e_2$$
$$= \{\xi \in C_2 : \xi = {}^1\xi e_1 + {}^2\xi e_2, ({}^1\xi, {}^2\xi) \in C(i_1) \times C(i_1)\}$$

$$C_2 = C(i_2) \times_e C(i_2) = C(i_2)e_1 + C(i_2)e_2$$
$$= \{\xi \in C_2 : \xi = \xi_1 e_1 + \xi_2 e_2, (\xi_1, \xi_2) \in C(i_2) \times C(i_2)\}$$

1.3 Idempotent Representation of Bicomplex Numbers

(I) $C(i_1)$-idempotent representation of Bicomplex Number

A Bicomplex Number $\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4$ has only two representations in the form of $C(i_1)$

First representation

$\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4 = (x_1 + i_1x_2) + i_2(x_3 + i_1x_4) = z_1 + i_2z_2$, where $z_1, z_2 \in C(i_1)$

Second representation

$\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4 = (x_1 + i_1x_2) + i_1i_2(x_4 - i_1x_3) = \psi_1 + i_1i_2\psi_2$, where $\psi_1, \psi_2 \in C(i_1)$

Bothe the idempotent representation of $C(i_1)$ representation are same.

Throughout this paper $C(i_1)$-idempotent representation of Bicomplex Number is given by

$$\xi = z_1 + i_2z_2 = (z_1 - i_1z_2)e_1 + (z_1 + i_1z_2)e_2 = {}^1\xi e_1 + {}^2\xi e_2$$

(II) *$C(i_2)$ – idempotent representation of Bicomplex Number*

A Bicomplex Number $\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4$ *has only two representations in the form of* $C(i_2)$

First representation

$\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4 = (x_1 + i_2x_3) + i_1(x_2 + i_2x_4) = w_1 + i_1w_2$, where $w_1, w_2 \in C(i_2)$

Second representation

$\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4 = (x_1 + i_2x_3) + i_1i_2(x_4 - i_2x_2) = \chi_1 + i_1i_2\chi_2$, *where* $\chi_1, \chi_2 \in C(i_2)$

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Bothe the idempotent representation of $C(i_2)$ representation are same.

Throughout this paper $C(i_2)$-idempotent representation of Bicomplex Number is given by

$$\xi = (x_1 + i_2 x_3) + i_1(x_2 + i_2 x_4)$$
$$= w_1 + i_1 w_2 = (w_1 - i_2 w_2)e_1 + (w_1 + i_2 w_2)e_2 = \xi_1 e_1 + \xi_2 e_2$$

1.4 Conjugations of Bicomplex Numbers

- $i_1 - Conjugation$

$\xi^* = \bar{z}_1 + \bar{z}_2 i_2, \forall z_1, z_2 \in C(i_1)$, $\bar{z}_1$ and $\bar{z}_2$ are complex conjugate of z_1 and z_2.

- $i_2 - Conjugation$

$\xi^\# = z_1 - z_2 i_2 \ \forall z_1, z_2 \in C(i_1)$.

- $i_1 i_2 - Conjugation$

$\xi' = \bar{z}_1 - \bar{z}_2 i_2 \ \forall z_1, z_2 \in C(i_1)$

2. Bicomplex Matrix (cf. [A1] & [L1])

We denote $C_2^{m\times n} = \{A = [\xi_{ij}]_{m\times n} : \xi_{ij} \in C_2\}$ the set of m ×n matrices with bicomplex entries.

Let $A = [\xi_{ij}]_{m\times n} \in C_2^{m\times n} \Rightarrow \xi_{ij} \in C_2$

A Bicomplex matrix A of order $m\times n$ can be represented in following ways

$$A = [\xi_{ij}]_{m\times n} = [x_{ij} + i_1 y_{ij} + i_2 u_{ij} + i_1 i_2 v_{ij}]_{m\times n}$$
$$= [x_{ij}]_{m\times n} + i_1[y_{ij}]_{m\times n} + i_2[u_{ij}]_{m\times n} + i_1 i_2 [v_{ij}]_{m\times n}$$
$$= [z_{ij}]_{m\times n} + i_2[w_{ij}]_{m\times n}$$
$$= [p_{ij}]_{m\times n} + i_1[q_{ij}]_{m\times n}$$
$$= [r_{ij}]_{m\times n} + i_1[s_{ij}]_{m\times n}$$
$$= [\alpha_{ij}]_{m\times n} + i_2[\beta_{ij}]_{m\times n}$$
$$= [\mu_{ij}]_{m\times n} + i_1 i_2[\lambda_{ij}]_{m\times n}$$
$$= [\phi_{ij}]_{m\times n} + i_1 i_2[\psi_{ij}]_{m\times n}$$

$C(i_1)$ - Idempotent representation

$$A = [\xi_{ij}]_{m\times n} = [{}^1\xi_{ij}]_{m\times n} e_1 + [{}^2\xi_{ij}]_{m\times n} e_2 = {}^1A e_1 + {}^2A e_2$$

$C(i_2)$ - Idempotent representation

$$A = [\xi_{ij}]_{m\times n} = [\xi_{1,ij}]_{m\times n} e_1 + [\xi_{2,ij}]_{m\times n} e_2 = A_1 e_1 + A_2 e_2$$

2.1 Conjugation of Bicomplex matrix:

There are three types of conjugation for each Bicomplex matrix $A = [\xi_{ij}]_{m\times n} \in C_2^{m\times n}$

- $i_1 - Conjugation$

$A^* = [\xi^*_{ij}]_{m\times n} = [\bar{z}_{ij} + i_2 \bar{w}_{ij}]_{m\times n}$

- $i_2 - Conjugation$

$A^\# = [\xi^\#_{ij}]_{m\times n} = [z_{ij} - i_2 w_{ij}]_{m\times n}$

- $i_1 i_2 - Conjugation$

$A' = [\xi'_{ij}]_{m\times n} = [\bar{z}_{ij} - i_2 \bar{w}_{ij}]_{m\times n}$

2.2 Transpose of Bicomplex Matrix:

Let $A = [\xi_{ij}]_{m\times n}$ be a Bicomplex Matrix of order $m\times n$

Then transpose of A is denoted by A^t and given by $A^t = [\xi_{ji}]_{n\times m}$ be a Bicomplex Matrix of order $n\times m$

- **Properties of Transpose Matrix**

(i) $(A^t)^t = A$

(ii) $(A+B)^t = A^t + B^t$

(iii) $(AB)^t = B^t A^t$

(iv) $(cA)^t = cA^t$

(v) $\det(A^t) = \det(A)$

(vi) $(A^t)^{-1} = (A^{-1})^t$

2.3 Symmetric Matrix:

A Bicomplex matrix $A = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ is said to be symmetric matrix if $A^t = A$.

2.4 Skew-Symmetric Matrix:

A Bicomplex matrix $A = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ is said to be Skew-Symmetric matrix if $A^t = -A$.

2.5 Transpose of Conjugate Matrix

- ***Transpose of $i_1 -$ Conjugate*** **Matrix (*$i_1 -$ Tranjugate*** **Matrix)**

$(A^t)^* = (A^*)^t = [\xi^*_{ji}]_{n\times m}$

- ***Transpose of $i_2 -$ Conjugate*** **Matrix (*$i_2 -$ Tranjugate*** **Matrix)**

$(A^t)^\# = (A^\#)^t = [\xi^\#_{ji}]_{n\times m}$

- **Transpose of *$i_1 i_2 -$ Conjugate Matrix* (*$i_1 i_2 -$ Tranjugate*** **Matrix)**

$(A^t)' = (A')^t = [\xi'_{ji}]_{n\times m}$

2.6 Bicomplex Hermitian Matrix:

- ***$i_1 -$ Hermitian*** **Matrix**

A Bicomplex matrix $A = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ is said to be $i_1 - Hermitian$ Matrix if

$(A^t)^* = (A^*)^t = A$

Note 2.1: If A is $i_1 - Hermitian$ Matrix then $A^* = A^t$

- ***$i_2 -$ Hermitian*** **Matrix**

A Bicomplex matrix $A = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ is said to be $i_2 - Hermitian$ Matrix if

$(A^t)^\# = (A^\#)^t = A$

Note 2.2: If A is $i_2 - Hermitian$ Matrix then $A^\# = A^t$

- ***$i_1 i_2 -$ Hermitian*** **Matrix**

A Bicomplex matrix $A=[\xi_{ij}]_{n\times n}\in C_2^{n\times n}$ is said to be $i_1\, i_2-$ *Hermitian* Matrix if $\left(A^t\right)^{'}=(A^{'})^t=A$

Note 2.3: If A is $i_1\, i_2-$ *Hermitian* Matrix then $A^{'}=A^t$

2.7 Bicomplex Skew- Hermitian Matrix

- $\boldsymbol{i_1-Skew-\ Hermitian}$ **Matrix**

  A Bicomplex matrix $A=[\xi_{ij}]_{n\times n}\in C_2^{n\times n}$ is said to be $i_1-Skew-Hermitian$ Matrix if $\left(A^t\right)^*=(A^*)^t=-A$

Note 2.4: If A is $i_1-Skew-Hermitian$ Matrix then $A^*=-A^t$

- $\boldsymbol{i_2-Skew-\ Hermitian}$ **Matrix**

  A Bicomplex matrix $A=[\xi_{ij}]_{n\times n}\in C_2^{n\times n}$ is said to be $i_2-Skew-Hermitian$ Matrix if $\left(A^t\right)^{\#}=(A^{\#})^t=-A$

Note 2.5: If A is $i_2-Skew-Hermitian$ Matrix then $A^{\#}=-A^t$

- $\boldsymbol{i_1\, i_2-Skew-Hermitian}$ **Matrix**

  A Bicomplex matrix $A=[\xi_{ij}]_{n\times n}\in C_2^{n\times n}$ is said to be $i_1\, i_2-Skew-$ *Hermitian* Matrix if $\left(A^t\right)^{'}=(A^{'})^t=-A$

Note 2.6: If A is $i_1\, i_2-Skew-Hermitian$ Matrix then $A^{'}=-A^t$

Properties **2.1 to 2.6** have given in **[A1]**. We have also proved these properties and get in different notations.

Properties of $\boldsymbol{i_1-Hermitian}$ Matrix 2.1:

(i) The matrix A is $i_1-Hermitian$ Matrix iff ${}^1A=\left({}^2A^t\right)^*$

(ii) The matrix A is $i_1-Hermitian$ Matrix iff $A_1=(A_2)^t$

(iii) All the $i_1-Hermitian$ Matrix are of the form

$$A={}^1A\,e_1+\left({}^1A^t\right)^*e_2=A_1\,e_1+(A_1)^t\,e_2$$

Proof:

(i) Let A is $i_1-Hermitian$ Matrix

i.e. $\left(A^t\right)^*=A$

$$\Leftrightarrow\left({}^1A^t\right)^*e_2+\left({}^2A^t\right)^*e_1={}^1A\,e_1+{}^2A\,e_2$$

$$\Leftrightarrow {}^1A=\left({}^2A^t\right)^* \text{ and } {}^2A=\left({}^1A^t\right)^*$$

$$\Leftrightarrow {}^1A=\left({}^2A^t\right)^*$$

(ii) Let A is $i_1-Hermitian$ Matrix

i.e. $\left(A^t\right)^*=A$

$$\Leftrightarrow(A_1)^t\,e_2+(A_2)^t\,e_1=A_1\,e_1+A_2\,e_2$$

$$\Leftrightarrow A_1=(A_2)^t \text{ and } A_2=(A_1)^t$$

$$\Leftrightarrow A_1=(A_2)^t$$

(iii) By using **(i)** and **(ii)**

$$A={}^1A\,e_1+\left({}^1A^t\right)^*e_2=A_1\,e_1+(A_1)^t\,e_2$$

Similarly, **Properties 2.2 to 2.6** can be proved.

Properties of $\boldsymbol{i_1-Skew-\ Hermitian}$ **Matrix 2.2:**

(i) The matrix A is $i_1-Skew-$ *Hermitian* Matrix iff ${}^1A=-\left({}^2A^t\right)^*$

(ii) The matrix A is $i_1-Skew-$ *Hermitian* Matrix iff $A_1=-(A_2)^t$

(iii) All the $i_1-Skew-\ Hermitian$ Matrix are of the form

$$A={}^1A\,e_1-\left({}^1A^t\right)^*e_2=A_1\,e_1-(A_1)^t\,e_2$$

Properties of $\boldsymbol{i_2-\ Hermitian}$ Matrix 2.3:

(i) The matrix A is $i_2-Hermitian$ Matrix iff ${}^1A={}^2A^t$

(ii) The matrix A is $i_2-Hermitian$ Matrix iff $A_1=(A_2^t)^{\#}$

(iii) All the $i_2-Hermitian$ Matrix are of the form

$$A={}^1A\,e_1+{}^1A^t\,e_2=A_1\,e_1+(A_1^t)^{\#}\,e_2$$

Properties of $\boldsymbol{i_2-Skew-\ Hermitian}$ **Matrix 2.4:**

(i) The matrix A is $i_2-Skew-$ *Hermitian* Matrix iff ${}^1A=-{}^2A^t$

(ii) The matrix A is $i_2-Skew-$ *Hermitian* Matrix iff $A_1=-(A_2^t)^{\#}$

(iii) All the $i_2-Skew-\ Hermitian$ Matrix are of the form

$$A={}^1A\,e_1-{}^1A^t\,e_2=A_1\,e_1-(A_1^t)^{\#}\,e_2$$

Properties of $\boldsymbol{i_1\, i_2-\ Hermitian}$ Matrix 2.5:

(i) The matrix A is $i_1\, i_2-Hermitian$ Matrix iff ${}^1A=({}^1A^t)^{'}$ and ${}^2A=({}^2A^t)^{'}$

(ii) The matrix A is $i_1\, i_2-Hermitian$ Matrix iff $A_1=(A_1^t)^{'}$ and $A_2=(A_2^t)^{'}$

(iii) All the $i_1\, i_2 - Hermitian$ Matrix are of the form

$$A = ({}^1A^t)' e_1 + ({}^2A^t)' e_2 = (A_1^t)' e_1 + (A_2^t)' e_2$$

Properties of $i_1\, i_2 - Skew - Hermitian$ Matrix 2.6:

(i) The matrix A is $i_1\, i_2 - Skew - Hermitian$ Matrix iff ${}^1A = -({}^1A^t)'$ and ${}^2A = -({}^2A^t)'$

(ii) The matrix A is $i_1\, i_2 - Skew - Hermitian$ Matrix iff $A_1 = -(A_1^t)'$ and $A_2 = -(A_2^t)'$

(iii) All the $i_1\, i_2 - Skew - Hermitian$ Matrix are of the form

$$A = -({}^1A^t)' e_1 - ({}^2A^t)' e_2 = -(A_1^t)' e_1 - (A_2^t)' e_2$$

Note 2.7:

Let $A = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ and $\xi_{ij} = z_{ij} + i_2 w_{ij}$

(i) If A is $i_1 - Hermitian$ Matrix, then $\xi_{ii} = z_{ii} + i_2 w_{ii} \in C(i_2)$

(ii) If A is $i_1 - Skew - Hermitian$ Matrix, then $\xi_{ii} \in i_1 C(i_2)$

(iii) *If A is* $i_2 -$ Hermitian *Matrix, then* $\xi_{ii} = z_{ii} \in C(i_1)$

(iv) *If A is* $i_2 -$ Skew $-$ Hermitian *Matrix, then* $\xi_{ii} = i_2 w_{ii} \in i_2 C(i_1)$

(v) *If A is* $i_1\, i_2 -$ Hermitian *Matrix, then* $\xi_{ii} \in H$

(vi) *If A is* $i_1\, i_2 -$ Skew $-$ Hermitian *Matrix, then* $\xi_{ii} \in i_1 H$ *or* $\xi_{ii} \in i_2 H$

Theorem 2.1: *If A is* $i_1 -$ Hermitian *Matrix then*

(i) i_1A *is* $i_1 -$ Skew $-$ Hermitian *Matrix*

(ii) i_2A *is* $i_1 -$ Hermitian *Matrix*

(iii) i_1i_2A *is* $i_1 -$ Skew $-$ Hermitian *Matrix*

Theorem 2.2: *If A is* $i_1 -$ Skew $-$ Hermitian *Matrix, then*

(i) i_1A *is* $i_1 -$ Hermitian *Matrix*

(ii) i_2A *is* $i_1 -$ Skew $-$ Hermitian *Matrix*

(iii) i_1i_2A *is* $i_1 -$ Hermitian *Matrix*

Theorem 2.3: *If A is* $i_2 -$ Hermitian *Matrix, then*

(i) i_1A *is* $i_2 -$ Hermitian *Matrix*

(ii) i_2A *is* $i_2 -$ Skew $-$ Hermitian *Matrix*

(iii) i_1i_2A *is* $i_2 -$ Skew $-$ Hermitian *Matrix*

Theorem 2.4: *If A is* $i_2 -$ Skew $-$ Hermitian *Matrix, then*

(i) i_1A *is* $i_2 -$ Skew $-$ Hermitian *Matrix*

(ii) i_2A *is* $i_2 -$ Hermitian *Matrix*

(iii) i_1i_2A *is* $i_2 -$ Hermitian *Matrix*

Theorem 2.5: *If A is* $i_1\, i_2 -$ Hermitian *Matrix, then*

(i) i_1A *is* $i_1\, i_2 -$ Skew $-$ Hermitian *Matrix*

(ii) i_2A *is* $i_1\, i_2 -$ Skew $-$ Hermitian *Matrix*

(iii) i_1i_2A *is* $i_1\, i_2 -$ Hermitian *Matrix*

Theorem 2.6: *If A is* $i_1\, i_2 -$ Skew $-$ Hermitian *Matrix then*

(i) i_1A *is* $i_1\, i_2 -$ Hermitian *Matrix*

(ii) i_2A *is* $i_1\, i_2 -$ Hermitian *Matrix*

(iii) i_1i_2A *is* $i_1\, i_2 -$ Skew $-$ Hermitian *Matrix*

Theorem 2.7:

(i) If A is $i_1 - Hermetian$ Matrix, then A^n is also $i_1 - Hermetian$ Matrix.

(ii) *If A is* $i_1 - Skew - Hermetian$ *Matrix, then* $(A^n)^{*t} = (-1)^n A^n$.

Corollary 2.1:

(i) If A is $i_1 - Hermetian$ Matrix, then $(i_1A)^n$ is $i_1 - Hermetian$ Matrix if n is even and $i_1 - Skew - Hermetian$ Matrix if n is odd.

(ii) *If A is* $i_1 -$ Skew $-$ Hermetian *Matrix, then* $(i_1A)^n$ *is* $i_1 -$ Hermetian *Matrix.*

Corollary 2.2:

(i) If A is $i_1 - Hermetian$ Matrix, then $(i_2A)^n$ is also $i_1 - Hermetian$ Matrix.

(ii) If A is $i_1 - Skew - Hermetian$ Matrix, then $(i_2A)^n$ is $i_1 - Hermetian$ Matrix if n is even and $i_1 - Skew - Hermetian$ Matrix if n is odd.

Corollary 2.3:

(i) If A is $i_1 - Hermetian$ Matrix, then $(i_1i_2A)^n$ is $i_1 - Hermetian$ Matrix if n is even and $i_1 - Skew - Hermetian$ Matrix if n is odd.

(ii) If A is $i_1 - Skew - Hermetian$ Matrix, then $(i_1i_2A)^n$ is $i_1 - Hermetian$ Matrix.

Theorem 2.8:

(i) If A is $i_2 - Hermetian$ Matrix, then A^n is also $i_2 - Hermetian$ Matrix.

(ii) *If A is* $i_2 - Skew - Hermetian$ *Matrix, then* $(A^n)^{\#t} = (-1)^n A$.

Corollary 2.4:

i. If A is $i_2 - Hermetian$ Matrix, then $(i_1A)^n$ is also $i_2 - Hermetian$ Matrix.

ii. If A is $i_2 - Skew - Hermetian$ Matrix, then $(i_1A)^n$ is $i_2 - Hermetian$ Matrix if n iseven and $i_2 - Skew - Hermetian$ Matrix if n is odd.

Corollary 2.5:

i. If A is $i_2 - Hermetian$ Matrix, then $(i_2A)^n$ is $i_2 - Hermetian$ Matrix if n is even and $i_2 - Skew - Hermetian$ Matrix if n is odd. .

ii. If A is $i_2 - Skew - Hermetian$ Matrix, then $(i_2A)^n$ is $i_2 - Hermetian$ Matrix.

Corollary 2.6:

i. If A is $i_2 - Hermetian$ Matrix, then $(i_1i_2A)^n$ is $i_2 - Hermetian$ Matrix if n is even and $i_2 - Skew - Hermetian$ Matrix if n is odd.

ii. If A is $i_2 - Skew - Hermetian$ Matrix, then $(i_1i_2A)^n$ is $i_2 - Hermetian$ Matrix.

Theorem 2.9:

i. If A is $i_1\, i_2 - Hermetian$ Matrix, then A^n is also $i_1\, i_2 - Hermetian$ Matrix.

ii. *If A is* $\mathrm{i}_1\, \mathrm{i}_2 - \mathrm{Skew} - \mathrm{Hermetian}$ *Matrix, then* $(A^n)^{'t} = (-1)^n A$.

Corollary 2.7:

i. If A is $i_1\, i_2 - Hermetian$ Matrix, then $(i_1A)^n$ is $i_1\, i_2 - Hermetian$ Matrix if n is even and $i_1\, i_2 - Skew - Hermetian$ Matrix if n is odd.

ii. If A is $i_1 i_2 - Skew - Hermetian$ Matrix, then $(i_1A)^n$ is $i_1\, i_2 - Hermetian$ Matrix.

Corollary 2.8:

i. If A is $i_1\, i_2 - Hermetian$ Matrix, then $(i_2A)^n$ is $i_1\, i_2 - Hermetian$ Matrix if n is even and $i_1\, i_2 - Skew - Hermetian$ Matrix if n is odd.

ii. If A is $i_1 i_2 - Skew - Hermetian$ Matrix, then $(i_2A)^n$ is $i_1\, i_2 - Hermetian$ Matrix.

Corollary 2.9:

i. If A is $i_1\, i_2 - Hermetian$ Matrix, then $(i_1i_2A)^n$ is also $i_1\, i_2 - Hermetian$ Matrix.

ii. If A is $i_1 i_2 - Skew - Hermetian$ Matrix, then $(i_1i_2A)^n$ is $i_1\, i_2 - Hermetian$ Matrix if n is even and $i_1\, i_2 - Skew - Hermetian$ Matrix if n is odd.

Properties 2.7:

(i) A Bicomplex matrix $C = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ can uniquely be written as the sum of a i_1-Hermetian Matrix A and a i_1-Skew-Hermetian Matrix B.

(ii) .A Bicomplex matrix $C = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ can uniquely be written as the sum of a i_2-Hermetian Matrix A and a i_2-Skew-Hermetian Matrix B.

(iii) A Bicomplex matrix $C = [\xi_{ij}]_{n\times n} \in C_2^{n\times n}$ can uniquely be written as the sum of a i_1i_2 – Hermetian Matrix A and a i_1i_2 -Skew-Hermetian Matrix B.

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