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K.P.S. Baghel[1] * and M. Jain[2]

[1]Department of Mathematics, Rajkiya Mahavidhyalaya Targawan Jaithra, Etah (UP), India

[2]Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee (UK), India

Email: drkunwarpalsinghbaghel@gmail.com

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[1]Department of Mathematics, Rajkiya Mahavidhyalaya Targawan Jaithra, Etah (UP), India
[2]Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee (UK), India

Email: drkunwarpalsinghbaghel@gmail.com

ABSTRACT

The paper addresses the Markovian queueing problem for stochastic environments wherein repair jobs of failed machines arrive according to FCFS order. The repair facility consists of two heterogeneous repairmen who facilitate the repair of the failed machines with different repair rates. In case if the queue size is less than or equal to a control limit 'L', each failed machine is repaired by the both servers with effective at rates $\mu = \mu_1 + \mu_2$, where $\mu_1 > \mu_2$. When the queue size becomes greater than L and less than K, the machines are recovered with a faster rate μ_f. If the size of queue exceeds K, all the machines are repaired together in a batch at rate μ_b. To determine the steady-state probabilities and other key performance indices, a recursive technique is applied. To find the optimal value of the threshold parameter K, a cost function is also facilitated. The effects of system descriptors on the performance indices are visually depicted by graphs.

Keywords: *Markov, FCFS, Single and Batch service, Heterogeneous servers, Steady state, Recursive Technique, Cost Function.*

INTRODUCTION

Due to increase of population, the applicability of machines may be realized in any manufacturing company/ organization. Everyone has to face the crucial problems arisen in the daily life activities in the whole world. The machines play an important role to reduce the problems. For business function, one firm production manager contracts to another to supply its production quantity. In many industrial problems, machines are subject to random failures and not be renewed until the repair of first unit is completed. Sometimes it is possible that the arriving machines may be diverted due to long queue.

Several queue theorists presented their works in this direction [see: Elsayed (1981), Agnihothri, (1989). Lee et al. (1995), Hsieh and Wang (1995), Jain and Dhyani (1999), Choi and Lee (2001) and Wang and Ke (2003)]. Jain et al (2006) applied mean value analysis to study the machining problems. Yuan (2012) discussed k-out-of-N : G system wherein repairmen have multiple vacations. Yang and Wu (2015) provided cost minimization analysis of a vacation queue with N-policy and server breakdowns. Shekhar et al (2019) discussed modified Bessel series solution of the single server queueing model provision of feedback. Shekhar et al (2021) presented their views on optimal profit analysis for machining problem having finite number of operating machines (OM) as well as warm standby machines (WSM) under the supervision of a single unreliable server to repair in phases and organizational delay.

In the past, the mathematicians, who worked on bulk service are Bailey (1954), Borthakur (1971), Sim and Templeton (1985), Chaudhry et al. (1991), Babu Raj and Manoharan (1997), Jain et al. (2000), Kim and Choi (2003), Janssen and van Leeuwaarden (2005), Chakravarthy (2012). Singh et. al (2013), Sanjeet Singh and Naveen (2014). Chaudhary et. al (2019) studied $GI^X/Geo/c$ model to analyze simple and computational results. Shekhar et al (2020) applied bat algorithm to require optimal control of a service system having imergency vacation.

In this paper, we extend the work of Babu Raj and Manoharan by incorporating two servers with single and batch service. In this problem the rates are considered different with finite population. The whole paper is partitioned into following sections: In section 2, some assumptions with notations related to the model are given. Section 3 provides for governing equations wherein queue size distribution is obtained. In section 4, we attempt to find out the performance indices of the system. Numerical results are provided in section 5. The conclusion of the paper is suggested in the last section 6 that may be helpful for further researchers.

*Author for correspondence

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MODEL AND NOTATIONS

We consider a Markov queue with single and batch service and heterogeneous servers wherein the repair rates of the two servers are not identical. The tuple (n_1, n_2) defines the state of the system wherein n1, n2 $\geq$ 0 denotes the number of fault items in the queue.

The following assumptions are made to formulate the problem:

- The repair of failed operating machines which arrive in the queue is done in FCFS discipline. Two repairmen having different repair rates are facilitated to detect the fault of machines.
- The repair rate of server is μ_1 for the states (1,0) and (1,1) while μ_2 is the repair rate for the states (0, 1) and (1,1).
- When $2 \leq n \leq L$, each unit is repaired singly by both servers with repair rate $\mu = \mu_1 + \mu_2$ while the machine is repaired with faster rate μ_f in case of $L<n \leq K$.
- If the number of failed units exceed to K, the units are repaired in bulk/batch with repair rate μ_b.
- In case of repair completion, the repaired unit is treated as good as new one and it is used again in place of failed machine.
- If all the machines fail, the system is lost.

To develop the model, following notations are considered as:

M	-	The number of operating units
λ	-	Failure rate of units
μ_1	-	Repair rate of the primary server
μ_2	-	Repair rate of the secondary server
μ	-	Repair rate of the both servers
μ_f	-	Fast repair rate of the both servers
μ_b	-	Batch repair rate of the both servers

The Equations and Mathematical Analysis

The steady state balance equations governing the model are given by

$$M\lambda P_{0,0} = \mu_1 P_{1,0} + \mu_2 P_{0,1} + \mu_b \sum_{n=K+1}^{M-1} P_{n,1} \quad \ldots(1)$$

$$[(M-1)\lambda + \mu_1] P_{1,0} = M\lambda P_{0,0} + \mu_2 P_{1,1} \quad \ldots(2)$$

$$[(M-1)\lambda + \mu_2] P_{0,1} = \mu_1 P_{1,1} \quad \ldots(3)$$

$$[(M-2)\lambda + \mu] P_{1,1} = (M-1)\lambda P_{1,0} + (M-1)\lambda + \mu P_{2,1} \quad \ldots(4)$$

$$[(M-n-1)\lambda + \mu] P_{n,1} = (M-n)\alpha P_{n-1,1} + \mu P_{n+1,1}$$
$$2 \leq n \leq L-1 \quad \ldots(5)$$

$$[(M-L-1)\lambda + \mu] P_{L,1} = (M-L)\lambda P_{L-1,1} + \mu_f P_{L+1,1} \quad \ldots(6)$$

$$[(M-n-1)\lambda + \mu_f] P_{n,1} = (M-n)\lambda P_{n-1,1} + \mu_f P_{n+1,1}$$
$$L<n \leq K-1 \quad \ldots(7)$$

$$[(M-K-1)\lambda + \mu_f] P_{K,1} = (M-K)\lambda P_{K-1,1} \quad \ldots(8)$$

$$[(M-n-1)\lambda + \mu_b] P_{n,1} = (M-n)\lambda P_{n-1,1}$$
$$K<n \leq M-2 \quad \ldots(9)$$

$$\mu_b P_{M-1,1} = \lambda P_{M-2,1} \quad \ldots(10)$$

On solving equation (3), we find

$$P_{0,1} = \frac{\mu_1}{(M-1)\lambda + \mu_2} P_{1,1} \quad \ldots(11)$$

The equation (2) gives

$$P_{1,0} = \frac{M\lambda}{(M-1)+\mu_1} P_{0,0} + \frac{\mu_2}{(M-1)\lambda + \mu_1} P_{1,1} \quad \ldots(12)$$

Also equation (4) provides

$$P_{2,1} = \left[1 + \frac{(M-2)\lambda}{\mu} - \frac{(M-1)\lambda}{\mu}\left\{\frac{\mu_1}{(M-1)\lambda+\mu_2} + \frac{\mu_2}{(M-1)\lambda+\mu_1}\right\}\right] P_{1,1}$$
$$- \frac{M\lambda}{\mu} \frac{(M-1)\lambda}{[(M-1)\lambda + \mu_1]} P_{0,0} \quad \ldots(13)$$

Now, equations (5) gives

$$P_{n,1} = \left[1 + \sum_{i=3}^{n}\prod_{k=i}^{n} \frac{(M-k)\lambda}{\mu}\right] P_{2,1} - \frac{(M-1)\lambda}{\mu}\left[1 + \sum_{i=4}^{n}\prod_{k=i}^{n} \frac{(M-k)\lambda}{\mu}\right] P_{1,1}$$
$$3 \leq n \leq L \quad \ldots(14)$$

Also

$$P_{L+1,1} = \left[\frac{(M-L-1)\lambda + \mu}{\mu_f}\right] P_{L,1} - \frac{(M-L)\lambda}{\mu_f} P_{L-1,1} \quad \ldots(15)$$

Equation (6) provides

$$P_{L+2,1} = \left[\frac{(M-L-2)\lambda}{\mu_f}\right] P_{L+1,1} - \frac{(M-L-1)\lambda}{\mu_f} P_{L,1} \quad \ldots(16)$$

Using equation (6) in (7) , we find

$$P_{n,1} = \left[1 + \sum_{i=L+3}^{n}\prod_{k=i}^{n} \frac{(M-k)\lambda}{\mu}\right] P_{L+1,1} - \frac{(M-L-1)\lambda}{\mu_f}\left[1 + \sum_{i=L+2}^{n}\prod_{k=i}^{n} \frac{(M-k)\lambda}{\mu}\right] P_{L,1}$$
$$L+3 \leq n \leq K-1 \quad \ldots(17)$$

The equations (8), (9) and (10) provide solution as

$$P_{L,1} = \left[\frac{(M-L)\lambda}{(M-L-1)\lambda + \mu_f}\right] P_{L-1,1} \quad \ldots(18)$$

$$P_{n,1} = \prod_{i=K+1}^{n} \frac{(M-i)\lambda}{(M-i-1)\lambda+\mu_b} P_{K,1}$$

$K+1 \le n \le M-2$ …(19)

and

$$P_{M-1,1} = \frac{\lambda}{\mu_b} P_{Q-2,1} = \frac{\lambda}{\mu_b} \prod_{i=K+1}^{M-2} \frac{(M-i)\lambda}{(M-i-1)\lambda+\mu_b} P_{K,1}$$

…(20)

Now, using equation (1) and (19)-(20), we have the value of $P_{1,1}$ in terms of $P_{0,0}$

$$\sum_{n=0}^{1} P_{n,0} + \sum_{n=0}^{M-1} P_{n,1} = 1 \quad \text{…(21)}$$

The value of $P_{0,0}$ is obtained with the help of equation (21)

Some Performance Indices

Some different performance indices of the system are obtained as:

Throughput of the system is given by

$$TH = \mu_1 P_{1,0} + \mu_2 P_{0,1} + \mu \sum_{n=1}^{L} P_{n,1} + \mu_f \sum_{n=L+1}^{K} P_{n,1} + \mu_b \sum_{n=K+1}^{M-1} P_{n,1}$$

…(22)

Expected number of failed units in the system is given by

$$E(N) = P_{1,0} + P_{0,1} + \sum_{n=1}^{M-1} (n+1) P_{n,1} \quad \text{…(23)}$$

If there is no queue in the system or if the queue size is greater than K, the expected queue length is obtained by

$$L_q = \sum_{n=2}^{K} (n-1) P_{n,1} \quad \text{…(24)}$$

The expected total cost function is given by:

$$E(T_C) = C_0 P_{1,0} + C_1 \sum_{n=0}^{L} n P_{n,1} + C_f \sum_{n=L+1}^{K} P_{n,1} + C_b \sum_{n=K+1}^{M-1} P_{n,1}$$

…(25)

where,

C_0 -The service cost of one failed unit by primary server

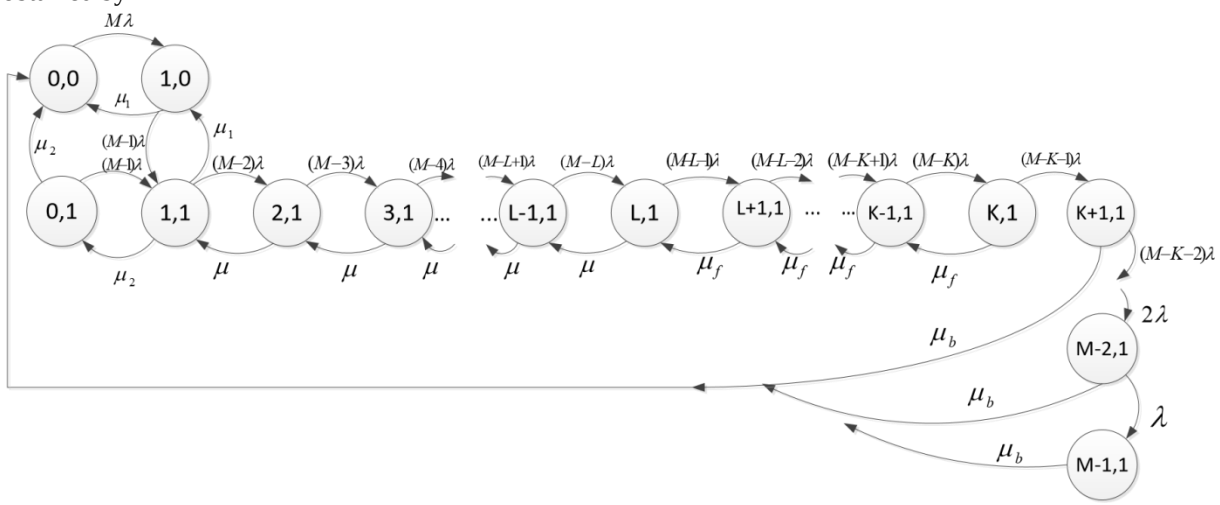
C_1 -The service cost of one failed unit in single service mode by secondary server with normal service rate

C_f - The service cost of one breakdown unit in single service mode by primary and secondary server with faster service rate

C_b- The service cost of a fault unit in batch service mode by primary and secondary server

NUMERICAL RESULTS

To draw the different graphs for this model, we assume the parameters as : M = 11 , μ_1 = 3 , μ_2 = 2 , $\mu(\mu_1 + \mu_2)$ =5 , μ_f = 8 & μ_b = 8. The authors consider different parameters to show the effect of the time on the performance indices. Figures 1, 2 & 3 depict time on the throughput of the system, expected queue length and expected number of failed machines in the figure at time t vs. failure rate λ. After looking after the figure 1, it is shown that the throughput increases for every failure rate (λ= 1,1.1,1.2). Further more from figures 2 & 3 , it is also noted that expected queue length and expected number of failed units in the system are also increase as time increases for every failure rate λ.



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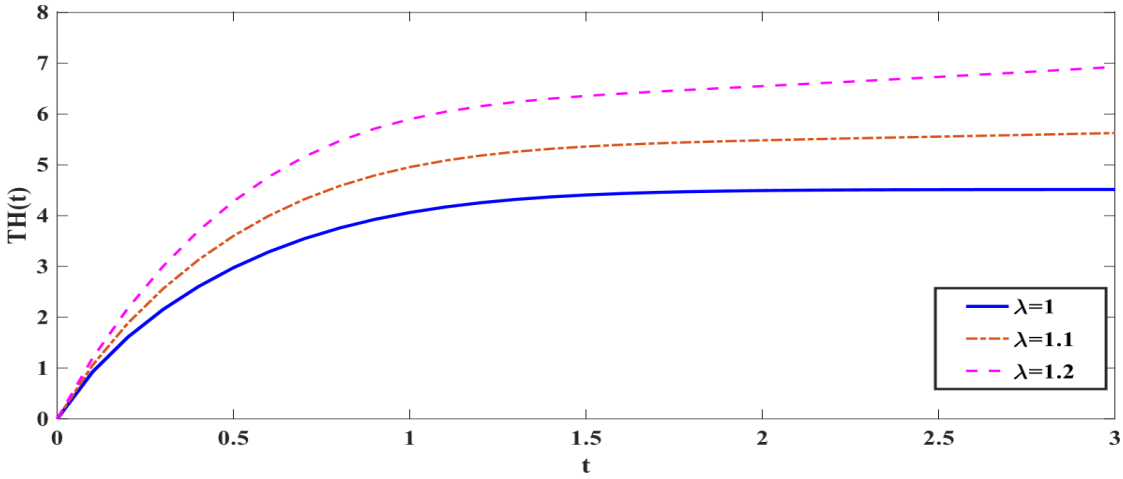

Fig. 1 : Throughput of the system vs. λ on varying t.

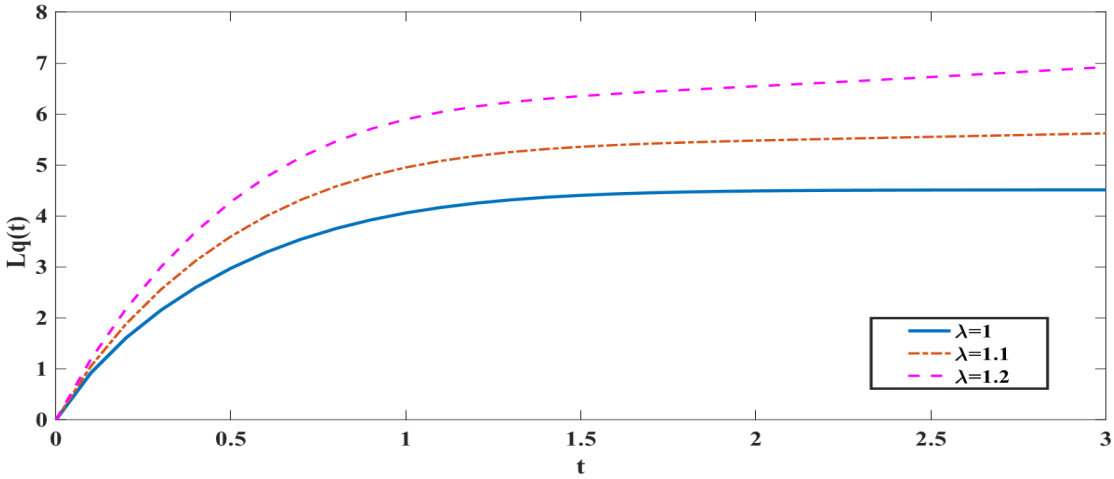

Fig. 2 : Expected queue length of the system vs. λ on varying t.

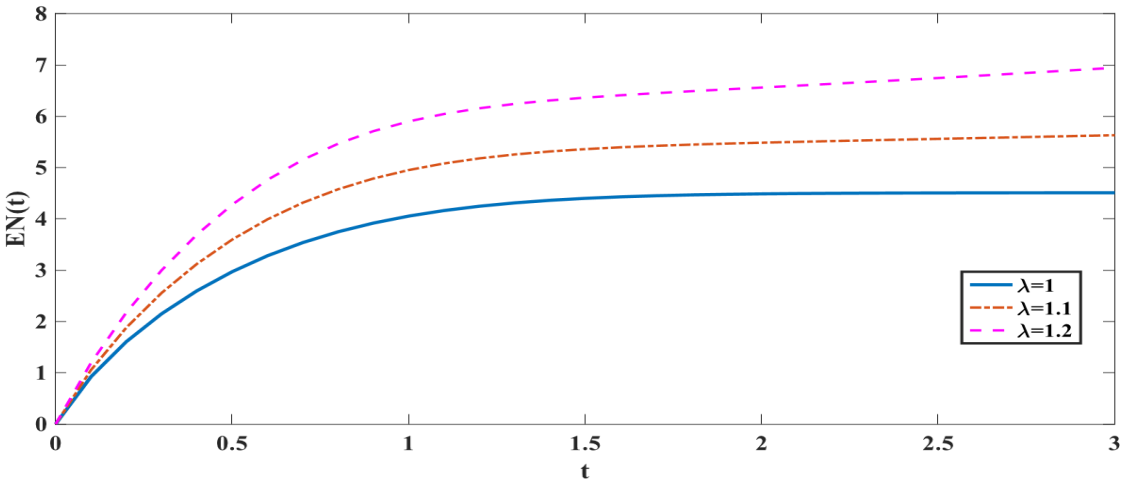

Fig. 3 : Expected number of fail machines vs. λ on varying t.

CONCLUSION

Our motto of the paper is to obtain the steady-state solution for heterogeneous servers Markov queue with single and batch service in explicit form. First, failed units are renewed in order of their failures. When the queue is too long to repair, the customers with machines may divert. Therefore, the service rate of server is increased. When the queue size is greater than K, the machines are repaired in batch so that desired goal may not be lost. A cost analysis is also suggested to find out the value of K. This model will be helpful to the production managers in industrial organization, management officers etc. who suffer for the maintenance of the system regarding the installation of the number of items and servers to continue the operation to look after a certain number of machine. The right number of repairmen is very important in the world of automation to grow out the wanted access of production in manufacturing company.

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