Vol. 3 Issue No.3, July-September 2021 e-ISSN 2456-7701
Journal of Science and Technological Researches
A Peer Reviewed Journal
Origin of Innovation
Domain: www.jstr.org.in, Email: editor@jstr.org.in

MODIFIED EXPONENTIAL ESTIMATOR UNDER STRATIFIED RANDOM SAMPLING USING MINIMUM AND MAXIMUM VALUES OF AUXILLIARY VARIABLE

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Date of Received	**27 July, 2021**
Date of Revised	**18 August, 2021**
Date of Acceptance	**01 September, 2021**
Date of Publication	**30 September, 2021**

DOI : https://doi.org/10.51514/JSTR.3.3.2021.33-39

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I-3 Vikas Nagar, Housing Board Colony, Berasia Road, Karond Bhopal-462038
Domain: www.jstr.org.in, Email: editor@jstr.org.in, Contact: 09713990647

 e-ISSN 2456-7701
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ABSTRACT

In this paper a modified ratio-type exponential estimator for estimating population mean under stratified random sampling using minimum and maximum values of auxiliary variable has been suggested. The mathematical expressions for the bias and mean square error have been derived up to first order of approximation. The numerical comparison has been also carried out which revealed that the proposed estimator performed better than other existing estimators.

Keywords: *Bias, Mean square error, Minimum and maximum values, Auxiliary variable, Stratified Rand om sampling, Exponential estimators*

INTRODUCTION

In sample surveys, auxiliary information is used at selection as well as estimation stages to improve the efficiency of estimators, a known auxiliary variable is always been the source of improvement in precision of estimates. A transformation can be used for the auxiliary variable to get even more efficient estimator when study variable Y is correlated with auxiliary variable X. To improve the efficiency of ratio estimator, introduced by Cochran (1940), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) suggested estimators for population mean by using information on higher order parameter which restrict their applicability. To overcome such restriction Mohanty and Sahoo (1995) introduced two estimators using Known information of minimum and maximum values of the auxiliary variable. Letter on, different authors modified the Mohanty and Sahoo (1995) estimators in order to improve their efficiency. In line with the modification of this estimators, Walia et al (2015) proposed two estimators for finite population mean using known information of minimum and maximum values of the auxiliary variable and coefficient of variation under simple random sampling. To increase the efficiency of ratio estimator, Bahl and Tuteja (1991) introduced ratio and product-type exponential estimators which was later extended by several authors including Singh and Audu (2015) Ahmed and Singh (2015). Hence, By combining the idea of Sahai and Ray (1980), Mohanty and Sahoo (1995), Solanki et al (2012) and Walia et al (2015) an estimator for finite population mean has been suggested by using minimum and maximum value of auxiliary variable under stratified random sampling.

2. Notation

Consider a finite Population of Size N is divided into L mutually exclusive strata of sizes N_h $(h=1,2,3......L)$ such that $\sum_{h=1}^{L} N_h = N$. Let Y_{hi} and X_{hi} be the values of the study and Auxiliary variable for i^{th} unit $(i=1,2,3,............N_h)$ in the h^{th} $(h=1,2,3,............L)$ stratum respectively. Also let a sample of size n_h $(h=1,2,3,............L)$ is drawn from h^{th} stratum independently by using Simple random sampling without replacement such that $\sum_{h=1}^{L} n_h = n$, where n is total sample size. Let y_{hi} and x_{hi} be the values of study and auxiliary variable respectively included in the sample at i^{th} draw $(i=1,2,3,............n_h)$ from h^{th} stratum.

$$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} Y_{hi} \quad , \quad \bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} X_{hi} \quad ,$$

$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ and $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$ be population and sample means of both study and auxiliary variable

 e-ISSN 2456-7701
Vol. 3 Issue No.3, July-September 2021

in h^{th} stratum.

$S_{hy}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(Y_{hi} - \bar{Y}_h\right)^2$ And

$S_{hx}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(X_{hi} - \bar{X}_h\right)^2$ be population mean square errors of study and auxiliary variable in h^{th} stratum.

$W_h = \frac{N_h}{N}$ is the stratum weight in the h^{th} stratum.

$f_h = \frac{n_h}{N_h}$ is the sampling fraction in the h^{th} stratum.

$R = \frac{\bar{Y}}{\bar{X}}$ is the population ratio.

$R_h = \frac{\bar{Y}_h}{\bar{X}_h}$ is the population ratio in the h^{th} stratum.

3. Existing Estimators

Stratified sample mean estimator $\hat{\bar{Y}}_{St} = \sum_{h=1}^{L} W_h \bar{y}_h$ is an unbiased estimator of population mean $\bar{Y}$ with variance $V(\hat{\bar{Y}}_{St}) = \sum_{h=1}^{L} W_h^2 \theta_h S_{Yh}^2$ where

$$\theta_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \quad \ldots(1)$$

Separate ratio estimator for single auxiliary variable is defined as

$$\hat{\bar{Y}}_{SR} = \sum_{h=1}^{L} W_h \frac{\bar{y}_h}{\bar{x}_h}\bar{X}_h \quad \ldots(2)$$

bias and MSE of $\hat{\bar{Y}}_{SR}$, to the first order of approximation, are

$$Bias(\hat{\bar{Y}}_{SR}) = \sum_{h=1}^{L} W_h \bar{Y}_h \theta_h \left(C_{Xh}^2 - \rho_{YXh} C_{Yh} C_{Xh}\right) \ldots(3)$$

$$MSE(\hat{\bar{Y}}_{SR}) = \sum_{h=1}^{L} W_h^2 Y_h^2 \theta_h \left(C_{Yh}^2 + C_{Xh}^2 - 2\rho_{YXh} C_{Yh} C_{Xh}\right) \quad (4)$$

Combined ratio estimator for population mean is defined as

$$\hat{\bar{Y}}_{CR} = \frac{\bar{y}_{St}}{\bar{x}_{St}}\bar{X} \quad \ldots(5)$$

Bias and MSE of $\hat{\bar{Y}}_{CR}$ up to the first order of approximation are given as

$$Bias(\hat{\bar{Y}}_{CR}) = \sum_{h=1}^{L} W_h \theta_h (S_{Xh}^2 - RS_{YXh}) \quad \ldots(6)$$

$$MSE(\hat{\bar{Y}}_{CR}) = \sum_{h=1}^{L} W_h^2 \theta_h (S_{Yh}^2 + R^2 S_{Xh}^2 - 2RS_{YXh}) \quad \ldots(7)$$

To improve the efficiency of ratio estimator and its applicability for population mean Mohanty and Sahoo (1995) suggested two estimators under simple random sampling using known information of minimum and maximum values of the auxiliary variable X given as:

$$t_{MS1} = \bar{y}\frac{\bar{V}}{\bar{v}} \quad \ldots(8)$$

and $$t_{MS2} = \bar{y}\frac{\bar{W}}{\bar{w}} \quad \ldots(9)$$

where $v_i = \frac{x_i + X_m}{X_M + X_m}$ and $w_i = \frac{x_i + X_M}{X_M + X_m}$

The expression for their biases and MSEs up to the first order of approximation are given as:

$$B\left(t_{MS1}\right) = \theta\bar{Y}\left(R_{MS1}^2 C_X^2 - R_{MS1}\rho_{YX} C_X C_Y\right) \ldots(10)$$

$$B\left(t_{MS2}\right) = \theta\bar{Y}\left(R_{MS2}^2 C_X^2 - R_{MS2}\rho_{YX} C_X C_Y\right) \ldots(11)$$

$$MSE\left(t_{MS1}\right) = \theta\bar{Y}^2\left(C_Y^2 - 2R_{MS1}\rho_{YX} C_X C_Y + R_{MS1}^2 C_X^2\right) \quad (12)$$

$$MSE\left(t_{MS2}\right) = \theta\bar{Y}^2\left(C_Y^2 - 2R_{MS2}\rho_{YX} C_X C_Y + R_{MS2}^2 C_X^2\right) \quad (13)$$

Where $R_{MS1} = \frac{\bar{X}}{\bar{X} + X_m}$ and $R_{MS2} = \frac{\bar{X}}{\bar{X} + X_M}$.

Using known information of minimum and maximum values of auxiliary variable Walia (2015) introduced two estimators which are ratio –type in nature are given as:

$$t_{W1} = \bar{y}\left(\frac{\bar{Z}}{\bar{z}}\right) \quad \ldots(14)$$

$$t_{W2} = \bar{y}\left(\frac{\bar{Z} + C_z}{\bar{z} + C_z}\right) \text{ Where } z_i = x_i + \frac{X_M}{X_m} \ldots (15)$$

The bias and MSE of the two estimators are given as:

$$B\left(t_{W1}\right) = \theta\bar{Y}\left(R_{W1}^2 C_X^2 - R_{W1}\rho_{YX} C_X C_Y\right) \ldots(16)$$

$$B\left(t_{W2}\right) = \theta\bar{Y}\left(R_{W2}^2 C_X^2 - R_{W2}\rho_{YX} C_X C_Y\right) \ldots(17)$$

$$MSE\left(t_{W1}\right)=\theta\bar{Y}^{2}\left(C_{Y}^{2}-2R_{W1}\rho_{YX}C_{X}C_{Y}+R_{W1}^{2}C_{X}^{2}\right) \quad \ldots(18)$$

$$MSE\left(t_{W2}\right)=\theta\bar{Y}^{2}\left(C_{Y}^{2}-2R_{W2}\rho_{YX}C_{X}C_{Y}+R_{W2}^{2}C_{X}^{2}\right) \quad (19)$$

where $R_{W1}=\frac{\bar{X}}{\bar{X}+\frac{X_M}{X_m}}$ and $R_{W2}=\frac{\bar{X}\left(\bar{X}+\frac{X_M}{X_m}\right)}{\left(\bar{X}+\frac{X_M}{X_m}\right)^{2}+S_x}$

4. Adopted Estimators

Using known information of minimum and maximum values of auxiliary variable, Mohanty and Sahoo (1995) estimators under stratified random sampling are adopted as:

$$t_{StMS1}=\sum_{h=1}^{L}W_h\bar{y}_h\left(\frac{\bar{V}_h}{\bar{v}_h}\right) \quad \ldots(20)$$

$$t_{StMS2}=\sum_{h=1}^{L}W_h\bar{y}_h\left(\frac{\bar{U}_h}{\bar{u}_h}\right) \quad \ldots(21)$$

Where $u_h=\frac{X_{hi}+X_{Mh}}{X_{Mh}+X_{mh}}$ and $v_h=\frac{X_{hi}+X_{mh}}{X_{Mh}+X_{mh}}$

The Biases and the MSE's of these estimators are given by

$$B(t_{StMS1})=\sum_{h=1}^{L}W_h\theta_h\bar{Y}\left(R_{StMS1h}^{2}C_{xh}^{2}-R_{StMS1h}\rho_{YXh}C_{Xh}C_{Yh}\right) \quad \ldots(22)$$

$$B(t_{StMS2})=\sum_{h=1}^{L}W_h\theta_h\bar{Y}\left(R_{StMS2h}^{2}C_{Xh}^{2}-R_{StMS2h}\rho_{YXh}C_{Xh}C_{Yh}\right) \quad \ldots(23)$$

$$MSE(t_{StMS1})=\sum_{h=1}^{L}W_h^{2}\theta_h\bar{Y}_h\left(C_{Yh}^{2}-2R_{StMS1h}\rho_{YXh}C_{Yh}C_{Xh}+R_{StMS1h}^{2}C_{Xh}^{2}\right) \quad \ldots(24)$$

$$MSE(t_{StMS2})=\sum_{h=1}^{L}W_h^{2}\theta_h\bar{Y}_h\left(C_{Yh}^{2}-2R_{StMS2h}\rho_{YXh}C_{Yh}C_{Xh}+R_{StMS2h}^{2}C_{Xh}^{2}\right) \quad \ldots(25)$$

Where $R_{StMS1h}=\frac{\bar{X}_h}{\bar{X}_h+X_{mh}}$ and $R_{StMS2h}=\frac{\bar{X}_h}{\bar{X}_h+X_{Mh}}$

Walia (2015) estimators under stratified sampling can also be defined as:

$$t_{StW1}=\sum_{h=1}^{L}W_h\bar{y}_h\left(\frac{\bar{Z}_h}{\bar{z}_h}\right) \quad \ldots(26)$$

$$t_{StW2}=\sum_{h=1}^{L}W_h\bar{y}_h\left(\frac{\bar{Z}_h+C_{zh}}{\bar{z}_h+C_{zh}}\right) \quad \ldots(27)$$

Where $z_h=X_{hi}+\frac{X_{Mh}}{X_{mh}}$ and $C_{zh}=\frac{S_{zh}}{\bar{Z}_h}=\frac{S_{zh}}{X_{hi}+\frac{X_{Mh}}{X_{mh}}}$

The Biases and MSEs of these estimators, up to first order approximation are given by

$$B(t_{StW1})=\sum_{h=1}^{L}W_h\theta_h\bar{Y}_h\left(R_{StW1h}^{2}C_{Xh}^{2}-R_{StW1h}\rho_{YXh}C_XC_Y\right) \quad \ldots(28)$$

$$B(t_{StW2})=\sum_{h=1}^{L}W_h\theta_h\bar{Y}_h\left(R_{StW2h}^{2}C_{Xh}^{2}-R_{StW2h}\rho_{YXh}C_XC_Y\right) \quad \ldots(29)$$

$$MSE(t_{StW1})=\sum_{h=1}^{L}W_h^{2}\theta_h\bar{Y}_h^{2}\left(C_{Yh}^{2}-2R_{StW1h}\rho_{YXh}C_XC_Y+R_{StW1h}^{2}C_{Xh}^{2}\right) \quad \ldots(30)$$

$$MSE(t_{StW2})=\sum_{h=1}^{L}W_h^{2}\theta_h\bar{Y}_h^{2}\left(C_{Yh}^{2}-2R_{StW2h}\rho_{YXh}C_XC_Y+R_{StW2h}^{2}C_{Xh}^{2}\right) \quad \ldots(31)$$

Where $R_{StW1h}=\frac{\bar{X}_h}{\bar{X}_h+\frac{X_{Mh}}{X_{mh}}}$ and $R_{StW2h}=\frac{\bar{X}_h\left(\bar{X}_h+\frac{X_{Mh}}{X_{mh}}\right)}{\left(\bar{X}_h+\frac{X_{Mh}}{X_{mh}}\right)+S_{Xh}}$

5. Proposed Estimator

A modified separate exponential type estimator for finite population mean of the study variable using minimum and maximum values of auxiliary variable X is proposed as:

$$t_P=\sum W_h\left[\bar{y}_h\left(2-\frac{\bar{x}_h}{\bar{X}_h}\right)^{\alpha_h}\exp\left(\frac{\bar{X}_h-\bar{x}_h}{(\bar{X}_h+\bar{x}_h)+2A_h}\right)\right] \quad \ldots(32)$$

where $A_h=X_{Mh}-X_{mh}$, and α_h is constant to be determined.

5.1 Bias and MSE of proposed estimator

Let $\bar{y}_h=\bar{Y}_h(1+e_{oh})$ and $\bar{x}_h=\bar{X}_h(1+e_{1h})$,

$$\begin{aligned}&\text{Therefore}, E(e_{oh})=E(e_{1h})=0\\&E(e_o^2)=\theta_hC_{Yh}^2, E(e_{1h}^2)=\theta_hC_{Xh}^2\\&E(e_oe_1)=\theta_h\rho_{YXh}C_{Yh}C_{Xh}\end{aligned} \quad (33)$$

To obtain bias and mean square error, let us substitute $\bar{y}_h$ and $\bar{x}_h$ in equation (32), therefore

$$t_P=\sum_{h=1}^{L}W_h\left[\bar{Y}_h(1+e_{oh})\left(2-\frac{\bar{X}_h(1+e_{1h})}{\bar{X}_h}\right)^{\alpha_h}\exp\left(\frac{\bar{X}_h-\bar{X}_h(1+e_{1h})}{(\bar{X}_h+\bar{X}_h(1+e_{1h}))+2A_h}\right)\right]$$

$$t_P=\sum_{h=1}^{L}W_h\bar{Y}_h\left[(1+e_{oh})\left(1-e_{1h}\right)^{\alpha_h}\exp\left(\frac{-\bar{X}_h e_{1h}}{2(\bar{X}_h+A_h)+\left(1+f_h\frac{e_{1h}}{2}\right)}\right)\right]$$

$$t_P=\sum_{h=1}^{L}W_h\bar{Y}_h\left[(1+e_{oh})\left(1-e_{1h}\right)^{\alpha_h}\exp\left(-\frac{f_h e_{1h}}{2}\left(1-\frac{f_h e_{1h}}{2}+\frac{f_h^2e^2{}_{1h}}{4}+.......\right)\right)\right] \text{ where } f_h=\frac{\bar{X}_h}{\bar{X}_h+A_h}$$

$$t_P=\sum_{h=1}^{L}W_h\bar{Y}_h\left[(1+e_{oh})\left(1-\alpha_h e_{1h}+\frac{\alpha_h(\alpha_h-1)e^2{}_{1h}}{2}........\right)\left(1-\frac{f_h e_{1h}}{2}+\frac{f_h^2e^2{}_{1h}}{4}+\frac{f_h^2e^2{}_{1h}}{8}.......\right)\right]$$

$$t_P=\sum_{h=1}^{L}W_h\bar{Y}_h\left[1-\frac{f_h e_{1h}}{2}+\frac{3f_h^2e^2{}_{1h}}{8}-\alpha_h e_{1h}+\frac{\alpha_h e^2{}_{1h}f_h}{2}+\frac{\alpha_h(\alpha_h-1)e^2{}_{1h}}{2}+e_{oh}-\frac{f_h e_{oh}e_{1h}}{2}-\alpha_h e_{oh}e_{1h}\right]$$

$$t_P-\bar{Y}=\sum_{h=1}^{L}W_h\bar{Y}_h\left[e_{oh}-e_{1h}\left(\alpha_h+\frac{f_h}{2}\right)+e^2{}_{1h}\left(\frac{\alpha_h f_h}{2}+\frac{3f_h^2}{8}+\frac{\alpha_h(\alpha_h-1)}{2}\right)-e_{oh}e_{1h}\left(\alpha_h+\frac{f_h}{2}\right)\right]$$

Taking expectation both the sides and using (33), Bias of t_P is obtained as

$$B(t_P)=\sum_{h=1}^{L}W_h\bar{Y}_h\left[\left(\frac{\alpha_h f_h}{2}+\frac{3f_h^2}{8}+\frac{\alpha_h(\alpha_h-1)}{2}\right)C_{Xh}^2-\left(\alpha_h+\frac{f_h}{2}\right)\rho_{YXh}C_{Yh}C_{Xh}\right] \quad ...(34)$$

MSE of the proposed estimator is obtained as

$$MSE\left(t_P\right)=E\left(t_P-\bar{Y}\right)^2=E\left[\sum_{h=1}^{L}W_h\bar{Y}_h\left\{e_{oh}-e_{1h}\left(\alpha_h+\frac{f_h}{2}\right)\right\}\right]^2$$

$$MSE\left(t_P\right)=\sum_{h=1}^{L}W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2+\frac{\left(2\alpha_h+f_h\right)^2}{4}C_{Xh}^2-\left(2\alpha_h+f_h\right)\rho_{YXh}C_{Yh}C_{Xh}\right] \quad ...(35)$$

To find the optimum value of α , differentiate (35) with respect to α_h and equate to 0, therefore,

$$\frac{\partial MSE\left(t_P\right)}{\partial\alpha}=0 \text{ which gives } \alpha_h=\frac{2\theta_h\rho_{YXh}C_{Yh}C_{Xh}-\theta_h C_{Xh}f_h}{2\theta_h C_{Xh}^2}$$

By substituting the value of α_h in (35) the minimum MSE of proposed estimator is given as

$$MSE(t_P)_{\min}=\sum_{h=1}^{L}W_h^2\bar{Y}_h^2\theta_h C_{Yh}^2\left(1-\rho_h^2\right) \quad ...(36)$$

Which is the MSE of the separate linear regression estimator.

6. Theoretical Comparison

In this section theoretical comparisons of the proposed estimator has been carried out with the estimators considered for study in literature and conditions have been found under which the proposed estimator is more efficient than other estimators.

i) The proposed estimator t_P is more efficient than stratified sample mean estimator if and only if

$$V\left(\hat{\bar{Y}}_{St}\right)-MSE(t_P)>0$$

$$\sum_{h=1}^{L}W_h^2\bar{Y}_h^2\theta_h C_{Yh}^2-\sum_{h=1}^{L}W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2+\frac{1}{4}C_{Xh}^2\left(2\alpha_h+f_h\right)^2-\rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h+f_h\right)\right]>0$$

$$\sum_{h=1}^{L}W_h^2\bar{Y}_h^2\theta_h\left[\frac{1}{4}C_{Xh}^2\left(2\alpha_h+f_h\right)^2-\rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h+f_h\right)\right]<0 \quad ...(37)$$

Thus, proposed estimator is more efficient than stratified sample mean estimator if condition (37) holds.

ii) The proposed estimator t_P is more efficient than the classical separate ratio estimator if and only if $MSE(\hat{\bar{Y}}_{SR})-MSE(t_P)>0$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2 + C_{Xh}^2 - 2\rho_{YXh}C_{Yh}C_{Xh}\right] - \sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h \left[C_{Yh}^2 + \frac{1}{4}C_{Xh}^2\left(2\alpha_h + f_h\right)^2 - \rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h + f_h\right)\right] > 0$$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[\left\{\frac{1}{4}C_{Xh}^2\left(2\alpha_h + f_h\right)^2 - 1\right\} - \rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h + f_h - 2\right)\right] < 0 \qquad \ldots(38)$$

Thus, proposed estimator t_P is more efficient than the classical separate ratio estimator if condition (38) holds.

iii) The proposed estimator t_P is more efficient than the estimator $\bar{y}_{StMS1}$ if and only if $MSE(\bar{y}_{StMS1}) - MSE(t_P) > 0$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_Y^2 - 2R_{StMS1h}\rho_{YX}C_XC_Y + R_{StMS1h}^2C_X^2\right] - \sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2 + \frac{1}{4}C_{Xh}^2\left(2\alpha_h + f_h\right)^2 - \rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h + f_h\right)\right] > 0$$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[\begin{array}{l} C_{Xh}^2\left\{\frac{1}{4}\left(2\alpha_h + f_h\right)^2 - R_{StMS1h}^2\right\} \\ -\rho_{YXh}C_{Yh}C_{Xh}\left\{\left(2\alpha_h + f_h\right) - 2R_{StMS1h}\right\}\end{array}\right] < 0 \qquad \ldots(39)$$

Thus, proposed estimator is more efficient than the estimator $\bar{y}_{StMS1}$ if condition (39) hold.

iv) The proposed estimator t_P is more efficient than the estimator $\bar{y}_{StMS2}$ if and only if $MSE(\bar{y}_{StMS2}) - MSE(t_P) > 0$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_Y^2 - 2R_{StMS2h}\rho_{YX}C_XC_Y + R_{StMS2h}^2C_X^2\right] - \sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2 + \frac{1}{4}C_{Xh}^2\left(2\alpha_h + f_h\right)^2 - \rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h + f_h\right)\right] > 0$$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[\begin{array}{l} C_{Xh}^2\left\{\frac{1}{4}\left(2\alpha_h + f_h\right)^2 - R_{StMS2h}^2\right\} \\ -\rho_{YXh}C_{Yh}C_{Xh}\left\{\left(2\alpha_h + f_h\right) - 2R_{StMS2h}\right\}\end{array}\right] < 0 \qquad \ldots(40)$$

Thus, proposed estimator is more efficient than the estimator $\bar{y}_{StMS2}$ if condition (40) hold.

v) The proposed estimator t_P is more efficient than the Walia et al (2015) estimator under stratified sampling $(\bar{y}_{StW1})$ if and only if $MSE(\bar{y}_{StW1}) - MSE(t_P) > 0$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2 - 2R_{StW1h}\rho_{YXh} + R_{StW1h}^2C_{Xh}^2\right] - \sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h \left[C_{Yh}^2 + \frac{1}{4}C_{Xh}^2\left(2\alpha_h + f_h\right)^2 - \rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h + f_h\right)\right] > 0$$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[\begin{array}{l} C_{Xh}^2\left\{\frac{1}{4}\left(2\alpha_h + f_h\right)^2 - R_{StW1h}^2\right\} \\ -\rho_{YXh}C_{Xh}C_{Xh}\left\{\left(2\alpha_h + f_h\right) + 2R_{StW1h}\right\}\end{array}\right] < 0 \qquad (41)$$

Thus, proposed estimator is more efficient than the estimator $\bar{y}_{StW1}$ if condition (41) hold.

vi) The proposed estimator t_P will be more efficient than the Walia et al (2015) estimator under stratified sampling $\bar{y}_{StW2}$ if and only if $MSE(\bar{y}_{StW2}) - MSE(t_P) > 0$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_{Yh}^2 - 2R_{StW2h}\rho_{YXh} + R_{StW2h}^2C_{Xh}^2\right] - \sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h \left[C_{Yh}^2 + \frac{1}{4}C_{Xh}^2\left(2\alpha_h + f_h\right)^2 - \rho_{YXh}C_{Yh}C_{Xh}\left(2\alpha_h + f_h\right)\right] > 0$$

$$\sum_{h=1}^{L} W_h^2\bar{Y}_h^2\theta_h\left[C_{Xh}^2\left\{\frac{1}{4}\left(2\alpha_h + f_h\right)^2 - R_{StW2h}^2\right\} - \rho_{YXh}C_{Yh}C_{Xh}\left\{\left(2\alpha_h + f_h\right) + 2R_{StW2h}\right\}\right] < 0 \qquad \ldots(42)$$

Thus, proposed estimator is more efficient than the estimator $\bar{y}_{StW2}$ if condition (42) hold.

7. Numerical Comparison

In this section mean square error and relative efficiency of proposed estimator and the estimators considered in study have been calculated by taking three real life population data from Shoaib (2018).The population have been stratified by using proportional allocation method.

Table 1: Population data sets

Stratum No.	N_h	n_h	$\bar{X}_h$	$\bar{Y}_h$	S^2_{hX}	S^2_{hY}	S_{hYX}	$Y_{h_{\max}}$	$Y_{h_{\min}}$	$X_{h_{\max}}$	$X_{h_{\min}}$
POPULATION I Y: Juice quantity per cane (in grams) X: Weight of cane (in grams)											
1	6	3	366.67	135.00	2706.67	80.00	440.00	150	125	450	300
2	12	6	310.83	99.17	1881.06	226.52	618.93	135	80	410	260
3	7	3	317.14	80.71	2890.48	120.24	444.05	100	70	420	250
POPULATION II Y: Amount of pocket money spent by students(in rupees) X: Annual income of student's parents (in '000 rupees)											
1	4	2	92.50	925.00	851.00	50833.33	6266.70	1250	750	135	70
2	10	6	57.20	535.00	31.07	11694.44	486.67	700	350	66	50
3	13	7	38.00	303.85	35.00	9775.64	445.83	450	150	47	28
POPULATION III Y: Area of leaf (in sq. cm) X: Weight of leaf (in mg)											
1	12	6	103.42	25.752	133.900	40.157	67.507	36.61	17.76	123	84
2	13	7	110.92	28.940	66.244	30.334	41.034	41.07	21.00	130	101
3	14	7	104.29	25.77	154.990	46.628	82.082	39.06	16.07	129	81

Table 2: MSE and PRE of Estimators

Estimators	Population I		Population II		Population III	
	MSE	PRE	MSE	PRE	MSE	PRE
$\hat{\bar{Y}}_{St}$	6.9128	100	535.2878	100	0.9682	100
$\hat{\bar{Y}}_{SR}$	2.8387	243.5168	181.3534	295.1604	0.3584	270.1451
$\hat{\bar{Y}}_{Slr}$	1.2974	532.8121	125.5617	426.3219	0.1134	853.7919
t_{StMS1}	2.2155	312.0242	206.3732	259.3784	0.5862	165.1655
t_{StMS2}	2.7055	255.5068	257.6865	207.7283	0.6460	149.8762
t_{StW1}	2.8124	245.7953	179.0670	298.9291	0.3643	265.7700
t_{StW2}	2.8099	246.016	178.5577	299.7818	0.3647	265.4785
t_P	1.2974	532.8121	125.5617	426.3219	0.1134	853.7919

CONCLUSION

From numerical illustration it is observed that the proposed estimator has minimum mean square error than all other estimators under study for all the three data sets. The mean square error of proposed estimator is same as separate linear regression estimator. This indicates that the proposed estimator is better than the estimators under study. In the recent past, many authors suggested efficient separate and combined ratio-type estimators by using the known value of population parameters of auxiliary variable which sometimes may not be available. In such situation when the maximum and minimum value of the auxiliary variable are available the proposed estimator will be useful and as efficient as separate linear regression estimator.

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